

Numbers, Quantities, and Units

This semester you often will be required to perform calculations on quizzes and examinations resulting in numeric answers to questions. The numeric response is the only correct answer to such a question, and no partial credit is awarded even if an incorrect value results from a simple arithmetic error (e.g., slip of a decimal point). The faculty adheres to this policy because in health care, such seemingly trivial errors often result in dire consequences. It has also always been the policy that no credit is given for answers that lack units altogether, or for values reported with incorrect units, since failure to supply proper units demonstrates a misunderstanding of the quantity being reported.

The purpose of this handout is to review several basic concepts related to the meaning and use of units, a subject that has caused some students trouble in the past. For most of you, the material covered will be old ground. Pay particular attention, however, to the sections (below) discussing Standard Scaling Prefixes, as well as Common Pitfalls.

Numbers and quantities

Numbers are abstract symbols that, by themselves, have no real physical meaning. In order to use numbers to quantify a physical property or entity (e.g., pressure, flow, mass, concentration), the dimensions of the unit quantity must be specified, and the number and its units must be treated as a single entity. For example, everyone knows that

$$1 = 2$$

is untrue. However, the equation

$$1 \text{ quart} = 2 \text{ pints}$$

is true because the quantities (volumes) represented by both sides of the equation are equal, even though the numbers themselves are not. By specifying the units, we relate the numbers to real physical quantities.

Computing with units

There are several basic rules that must be followed when working with units:

1. When two quantities are added or subtracted, both terms must have the same units, and the units of the result are the same as those of the operands. For example,

$$5 \text{ feet} + 2 \text{ feet} = 7 \text{ feet}$$

If the two quantities do not have the same units, the units must be converted before the quantities are added or subtracted.

2. When a dimensioned quantity is scaled by multiplying or dividing by a number without units, the units of the result are the same as those of the dimensioned operand. For example,

$$5 \times 2 \text{ feet} = 10 \text{ feet}$$

3. When two dimensioned quantities are multiplied (or divided), the units of the result are equal to the product (or quotient) of the units of the operands. For example,

$$5 \text{ feet} \times 2 \text{ feet} = 10 \text{ feet}^2$$

$$5 \text{ miles} / 2 \text{ hours} = 2.5 \text{ miles/hour}$$

$$5 \text{ miles} \times 2 \text{ hours} = 10 \text{ mile} \cdot \text{hours}$$

When the same dimensions appear in both numerator and denominator of the units, these dimensions can be factored out (canceled out). This simplifies the units of the results. For example,

$$5 \text{ moles} / \text{liter} \times 2 \text{ liters} / \text{min} = \frac{10 \text{ moles} \cdot \text{liters}}{\text{liter} \cdot \text{min}} = 10 \text{ moles} / \text{min}$$

4. When forming a ratio (or a percentage) of two quantities, both the numerator and denominator must have the same units. For example,

$$0.4 = \frac{2 \text{ meters}}{5 \text{ meters}} \quad \text{or} \quad 40\% = \frac{2 \text{ meters}}{5 \text{ meters}} \times 100$$

Note that the ratio has no units (it is dimensionless).

Unit conversion

Often it will be necessary to convert from one kind of unit to another. The easiest way to do this is to begin with the definition of the relationship between the units. For example, the equation

$$3 \text{ feet} = 1 \text{ yard}$$

defines the relationship between the units foot and yard as measures of length. Because both sides of this equation are equal, division of both sides by 3 feet makes the result equal to one, i.e.,

$$1 = 1 \text{ yard} / 3 \text{ feet}$$

or,

$$1 = \frac{1}{3} \text{ yard} / \text{foot}$$

The right-hand side of this equation is the conversion factor that is used to convert feet to yards. Because the conversion factor is equal to one, we can multiply any dimensioned quantity by the

conversion factor and not change the magnitude of the physical entity represented by the dimensioned number. To make the conversion, the quantity in feet is multiplied by the conversion factor, i.e.,

$$9 \text{ feet} \times \frac{1}{3} \text{ yard / foot} = \frac{9 \text{ yard} \cdot \text{feet}}{3 \text{ feet}} = 3 \text{ yards}$$

Note that “feet” factored out of the resultant units. Also, it is important to remember that the conversion factor merely changes the number in such a way that it fits the new units; the physical quantity represented is not changed (9 feet is just as long as 3 yards).

At times, multiple conversions must be made, and the easiest way to do this is to make one conversion at a time. For example, to convert 5 miles per hour to feet per minute, begin with the fundamental definitions,

$$1 \text{ hour} = 60 \text{ min}$$

$$1 \text{ mile} = 5280 \text{ feet}$$

Next, form the conversion factors,

$$1 = \frac{1}{60} \text{ hour / min}$$

$$1 = 5280 \text{ feet/mile}$$

Next multiply by one of the factors, and factor out the old units,

$$5 \text{ miles / hour} \times \frac{1}{60} \text{ hours / min} = \frac{5 \text{ miles} \cdot \text{hours}}{60 \text{ hour} \cdot \text{min}} = \frac{5}{60} \text{ miles / min} = \frac{1}{12} \text{ miles / min}$$

Then, multiply this result by the other conversion factor,

$$\frac{1}{12} \text{ miles / min} \times 5280 \text{ feet / mile} = 440 \frac{\text{miles} \cdot \text{feet}}{\text{min} \cdot \text{mile}} = 440 \text{ feet / min}$$

Of course, the conversion factors could have been multiplied first to form a single conversion factor, i.e.,

$$\frac{1}{60} \text{ hour / min} \times 5280 \text{ feet / mile} = 88 \frac{\text{hour} \cdot \text{feet}}{\text{min} \cdot \text{mile}}$$

Then, only a single multiplication would be needed, i.e.,

$$5 \text{ miles / hour} \times 88 \frac{\text{hour} \cdot \text{feet}}{\text{min} \cdot \text{mile}} = 440 \text{ feet / min}$$

Standard scaling prefixes

The size of a standard dimension can be changed by adding a prefix which indicates the relationship to the fundamental unit. This is very handy when dealing with very small or very large quantities. The most commonly used prefixes that you should know are listed below (note the prefix abbreviations).

Fundamental unit: gram

1 pico-gram	=	1 pg	=	1×10^{-12}	gram (one trillionth of a gram)
1 nano-gram	=	1 ng	=	1×10^{-9}	gram (one billionth of a gram)
1 micro-gram	=	1 μ g	=	1×10^{-6}	gram (one millionth of a gram)
1 milli-gram	=	1 mg	=	1×10^{-3}	gram (one thousandth of a gram)
1 centi-gram	=	1 cg	=	1×10^{-2}	gram (one hundredth of a gram)
1 deci-gram	=	1 dg	=	1×10^{-1}	gram (one tenth of a gram)
1 gram	=	1 g	=	1×10^0	gram
1 kilo-gram	=	1 Kg	=	1×10^3	gram (one thousand grams)
1 mega-gram	=	1 Mg	=	1×10^6	gram (one million grams)
1 giga-gram	=	1 Gg	=	1×10^9	gram (one billion grams)

Hints for working with units

1. Carry the units through in each step of a calculation.
2. Convert all necessary quantities to consistent units before starting the calculation. That is, convert all lengths to feet, for example, and convert all times to minutes, convert all masses to grams, etc.
3. Always make sure that the answers and the units are physically reasonable. For example, if you end up with units of (feet)² for a length, you blew it! Also, common sense can often alert you that you have made an arithmetic error, or used a formula incorrectly. Here are some (humorous) examples actually observed in the past: heart rate of 70 beats/second (about 60-times faster than normal); urine production rate of 4 liters/minute (more than a gallon of urine produced in less than a minute); a rat with 70 liters of body water (a rat that weighs in excess of 150 pounds); heart blood flow of 5 liters/minute with skin blood flow of 20 liters/minute (four-times more blood flowing through the skin than is flowing through the heart); body temperature of 101°C (temperature exceeding that of boiling water); a blood pressure of 100 mV (milli-volts, a unit of voltage not pressure); blood plasma pH of 5.5 (reasonable value for urine pH, but not plasma—okay, so you didn't already know this, but memorize the following fact since it will come up time and again in the course: plasma pH outside the range of 7 – 8 is incompatible with life!).

Common pitfalls

In the past, we have noticed several recurrent problems students experience in working with, and reporting, numeric quantities. We list the most common ones here.

Problems with the letter “m”. The letter “m” is used both as a standard prefix (milli= 10^{-3}), as well as the standard abbreviation for a unit of length (meter). In both cases, the letter is written in lower case. In the case of the milli prefix, examples include mL (milli-liter), mg (milli-gram), and mV (millivolt). In the case of the abbreviation for meter, examples include Km (kilometer), and cm (centimeter). Uppercase “M” is the standard abbreviation for molar concentration (moles/liter). Be careful, therefore, of the following: “mm” means millimeter, and “mM” means millimolar. “M” or “m” is not an abbreviation for minute.

Problems with “mole” and “molar”. One mole of a solute is an amount (mass) equivalent to the molecular weight of the solute in grams. There is no agreed upon abbreviation for mole, and as such, it should be spelled out in its entirety. For example, we might ask you to compute a flow of solute (amount per unit time), with the correct answer being 2 mmole/sec. Incorrect responses for this flow include 2 mm/sec, as well as 2 mM/sec.

Common abbreviations. Liter, l or L; second, s or sec; Siemen, S (unit of electrical conductance); Ohm, Ω (unit of electrical resistance); minute, min; volt, V; molar, M; meter, m; osmolar (osmole per liter), Osm (although some texts are sloppy and use Osm/L as well); gram, g or gm; millimeters of mercury, mmHg or torr (unit of hydrostatic pressure); equivalents, eq or equ. Pay special attention to the case of the letters (upper versus lower). If you are unsure of the unit’s abbreviation, then simply spell it out in its entirety (e.g., for “M”, you could report “molar”, “moles/L”, “moles/liter”, etc.).

“Feeling” for quantities. We ask you to always look at the result of a calculation, and attempt to decide whether the result is reasonable or not (see above). For example, if you compute a total blood volume of 5 mL for an adult, you should immediately realize that you have made a serious error; 5 L is much more reasonable (note that 5 mL is equivalent to about a teaspoon of fluid). For those of you who still think in British units, the following may be helpful for getting a general feeling for different quantities: 1 meter is a bit over a yard in length; 1 cm is a bit under half an inch; 1 liter (equal to 1000 cm^3) is a bit more than a quart; 1 fluid ounce is a bit under 30 mL; 1 liter of water weighs 1 Kg (a bit over 2 pounds), hence for water, 1 mL = 1 cm^3 = 1 gram; body fluids are dilute aqueous solutions and their volumes weigh about the same as pure water (if you empty a full bladder containing 500 mL, you immediately lose a bit over 1 pound of body weight).

Amounts, volumes, concentrations and flows. Students often incorrectly report amounts or volumes, when they intend to mean concentrations or flows (and vice versa). Amounts are expressed as weights (e.g., g, mg, μg), moles or equivalents (equ), and sometimes in so-called standard units of activity (U) or units of radioactivity (counts/min, Curies, etc.). Amounts of fluids in the body are expressed by their respective volume (e.g., mL, L). Concentrations are always expressed as an amount per unit volume (e.g., mg/L, mg/mL, mmole/L = mM), or rarely as a weight-percent (e.g., mg%, stated “milligrams percent,” equivalent to mg/100 mL or mg/dL). Flows of solutes in the body are expressed as an amount per unit time (e.g., mg/sec, moles/min). Flows of fluid are expressed as a volume of fluid per unit time (e.g., L/min, mL/sec).

Significant digits. No more than 3 significant digits are needed in calculations in this course. Note that significant digits does not mean the number of digits to the right of a decimal point, but rather, the number of digits in the mantissa when the value is expressed in scientific notation. For example, the number 0.000387 is only expressed to 3-significant-digit accuracy, since it can be represented in

scientific notation as 3.87×10^{-4} . Frequent use of scientific notation dramatically reduces the chance of numerical errors, by avoiding long lines of zeroes.

Study problems

1. If 5 steel rods, each 5 inches long, are welded together end-to-end, and then the resultant piece is connected to a 5-meter-long wooden rod, how long is the resultant structure in centimeters? (Hint: 2.54 cm = 1 inch.)
2. A box measures 5 inches by 3 millimeters by 5 miles. What is the surface area of the box in square centimeters and the volume in cubic meters? (Hint: keeping in mind three-significant-digit accuracy will simplify the calculation slightly.)
3. What is a flow of 10 nanoliters/minute equal to when expressed in milliquarts/day. (Hint: 1 liter = 1.06 quarts.)
4. A 150-pound college student, despondent over just breaking up with his girl friend, goes over to his friend's house for "moral support." His buddy hands him a full bottle of Wild Turkey (101 proof) and an 8-ounce tumbler, whereby the guy fills the glass to the rim and immediately downs it (in an attempt to drown his sorrows). How drunk does the student get? In other words, what is his peak percent blood alcohol level? (Additional information: the bourbon is rapidly absorbed and the alcohol evenly distributes in all body fluids, which weigh roughly 60% of total body weight; "101 proof" means that 50.5% of the bourbon is pure alcohol; there are 32 ounces/quart and 2.20 pounds/Kg; "percent alcohol content" is the fraction of alcohol in the body fluids expressed as a percent, or how many mL of alcohol per 100 mL of total fluid.)
5. The intracellular concentration of Na^+ in a spherical cell with radius $r = 10 \mu\text{m}$ equals 10 mM. What is the cell's volume, and how much sodium is contained in the cell? (Hint: cell volume = $\frac{4}{3}\pi r^3$, and recall that concentration equals amount divided by the cell volume.)

Answers to study problems

1. Length = $(5 \times 5 \text{ inches}) \times (2.54 \text{ cm/inch}) + (5 \text{ m}) \times (1 \text{ cm}/0.01 \text{ m}) = 564 \text{ cm}$.
2. Let L (length) = 5 miles, W (width) = 5 inches, and H (height) = 3 mm. First, let's convert all lengths to, say, centimeters:

$$L = (5 \text{ miles}) \times (5280 \text{ feet/mile}) \times (12 \text{ inches/foot}) \times (2.54 \text{ cm/inch}) = 8.05 \times 10^5 \text{ cm}$$

$$W = (5 \text{ inches}) \times (2.54 \text{ cm/inch}) = 12.7 \text{ cm}$$

$$H = (3 \text{ mm}) \times (0.1 \text{ cm}/1 \text{ mm}) = 0.3 \text{ cm}$$

(above calculation would have been dramatically simplified if I had remembered that there are 1.61 Km per mile)

There are 6 sides to a box, so simply compute the areas of three different sides and double that result:

$$\begin{aligned}
 \text{Surface area} &= 2 \times (L \times W + L \times H + W \times H) \\
 &= 2 \times (8.05 \times 10^5 \times 12.7 + 8.05 \times 10^5 \times 0.3 + (12.7 \times 0.3)) \\
 &= 2 \times (1.02 \times 10^7 + 2.41 \times 10^5) \\
 &= 2.09 \times 10^7 \text{ cm}^2
 \end{aligned}$$

Ignore, affects eighth significant digit of final answer

$$\begin{aligned}
 \text{Volume} &= L \times W \times H \\
 &= (8.05 \times 10^5 \text{ cm} \times 12.7 \text{ cm} \times 0.3 \text{ cm}) \times (1 \text{ m} / 100 \text{ cm})^3 \\
 &= (3.07 \times 10^6 \text{ cm}^3) \times (1 \text{ m}^3 / 10^6 \text{ cm}^3) \\
 &= 3.07 \text{ m}^3
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \text{Flow} &= (10 \text{ nL/min}) \times (10^{-9} \text{ L} / 1 \text{ nL}) \times (1.06 \text{ qt} / 1 \text{ L}) \times (1 \text{ mqt} / 10^{-3} \text{ qt}) \times (60 \text{ min/hr}) \times (24 \text{ hr/day}) \\
 &= 0.0153 \text{ mqt/day.}
 \end{aligned}$$

or, by combining some of the units

$$\text{Flow} = (10 \text{ nL/min}) \times (1060 \text{ mqt} / 10^9 \text{ nL}) \times (1440 \text{ min/day}) = 0.0153 \text{ mqt/day.}$$

$$\begin{aligned}
 4. \quad \text{Volume of alcohol ingested} &= 0.505 \times (8 \text{ oz}) \times (1 \text{ qt} / 32 \text{ oz}) \times (1 \text{ L} / 1.06 \text{ qt}) = 0.119 \text{ L} \\
 \text{Total body fluid volume} &= 0.60 \times (150 \text{ lb}) \times (1 \text{ L} / 2.20 \text{ lb}) = 40.9 \text{ L} \\
 \text{Percent alcohol content} &= (0.119 / 40.9) \times 100 = 0.291\%
 \end{aligned}$$

Note: he is very intoxicated, certainly unconscious, but probably not dead! Respiratory depression, ultimately leading to complete cessation of breathing, occurs when alcohol levels approach about 0.5%.

$$\begin{aligned}
 5. \quad \text{Volume} &= (4/3) \times 3.14 \times (10 \text{ } \mu\text{m} \times 10^{-4} \text{ cm} / \mu\text{m})^3 = 4.19 \times 10^{-9} \text{ cm}^3 = 4.19 \times 10^{-9} \text{ mL} \\
 &= 4.19 \times 10^{-12} \text{ L} = 4.19 \text{ pL}
 \end{aligned}$$

$$\begin{aligned}
 \text{Amt. Na}^+ &= 10 \text{ mmole/L} \times (10^{-3} \text{ mole/mmole}) \times 4.19 \times 10^{-12} \text{ L} = 4.19 \times 10^{-14} \text{ mole} \\
 &= 0.0419 \text{ pmole.}
 \end{aligned}$$